SRI AKILANDESWARI WOMEN'S COLLEGE WANDIWASH

PARTIAL DIFFERENTIAL EQUATIONS

MRS. SARASWATHI G
ASSISTANT PROFESSOR
DEPARTMENT OF MATHEMATICS

SWAMY ABEDHANANDHA EDUCTIONAL TRUST, WANDIWASH

Contents

1)Introduction:

- Unknown multi-variable differential equations with their partial derivatives are referred to as partial differential equations. One particular example of this type of equation is an ordinary different equation. It can be challenging to come up with an application ar which partial differential equations do not have an impact. One of core subjects of applied analysis is this one.
- Carl Gustav Jacob Jacobi revived the symbol in 1841. The roman letter d, which indicates a full derivative, is substituted with the si ∂ to represent a partial derivative.

- The first system of partial differential equations ever recorded in fluid dynamics is provided by the Euler equations, which Leonhard Euler found more than 250 years ago. The limiting case of the incompressible Euler equations is really the well-known Navier-Stokes equations.
- Adrien-Marie Legendre (1786) invented the contemporary partial derivative notation, although he later gave it up.
- Johann Bernoulli wrote a description of the Separation of Variable method between 1694 and 1697.

2)Basic preliminaries:

Partial differential equation:

A partial differential equation is an equation that has one or mor
partial derivatives of an unknown function of two or more independ
variables. For the difference between partial derivatives and ordina
single-variable derivatives, utilize the swirly-d symbol, ∂, also referre
to as "del."

Types of partial differential equations:

The different types of PDE are:

1. First-Order Partial Differential Equation
2. Second-Order Partial Differential Equation
3. Quasi-Linear Partial Differential Equation
4. Homogeneous Partial differential Equation

Charpit’s method:

This equation is called Charpit’s equation.

Example:

Find the complete integral of

Solution:

Find p and q:

Now,

Substitute p and q and solving we get,

This is the required complete solution.

3)Classification of partial differential equations:

Let a second order partial differential equation in the function u o

the two independent variables is of the form.

Equation (1) is classified as elliptic, parabolic or hyperbolic dependin

Example:

(Elliptic)

(Hyperbolic)

(Parabolic)

4)Elliptic partial differential equation:

The elliptic partial differential equation results if
The Laplace equation is an example of an elliptic partial differential
equation.

The possible solutions of Laplace equation are

Finite plate with value given in X direction:

Apply condition (iv), we get

Example:

The boundary value problem governing the steady state temperatu
distribution in a flat, thin, square plate is given by

Find the steady-state temperature distribution in the plate.

Solution:

5)Parabolic partial differential equation:

A parabolic partial differential equation results if

The equation for heat conduction is an example of a parabolic partia differential equation.

One dimensional heat equation with the initial and boundary conditions:

The one dimensional heat equation is

Where is the temperature at time at a point of distance from the left end of the rod.

The boundary conditions are

Example:

6)Hyperbolic partial differential equation:

A hyperbolic partial differential equation results is

The wave equation is an example of a hyperbolic partial differentia equation.

Vibrating string with zero initial velocity:

The boundary and initial conditions of the deflection are

The suitable solution is

Example:

Solution:

Differentiate partially we get

Now,

Hence the solution is